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A new model for the non-steady-state probe method to measure thermal properties of porous media

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Abstract—The starting point is the modified Jaeger model. This cylindrical model takes into account the major properties of the probe, the thermal conductivity and the heat capacity of the medium and the contact resistance between the probe and the medium. Because the mathematical description results in non-linear functions of these three thermal parameters, the Gauss–Newton iteration method is used. To obtain a better description of the measurement for small time values, a second-order time correction is added. The experimental test results in an inaccuracy of 0.5–2% for the thermal conductivity, while the two other parameters are accurate to within 5–15%.

INTRODUCTION

IN THE non-steady-state probe method, a needle-shaped probe is embedded in the material to be examined. The probe consists of a heating wire and a temperature sensor, which are fitted into a long, thin tube. To hold the wires in position and to prevent electrical contact, the remaining space inside the tube is filled with an insulating material.

Characteristic of the non-steady-state probe method is the measurement of the temperature response of the probe on a suddenly changing energy dissipation in the probe. This response is a function of the dimensions and the thermal properties of the probe, the thermal properties of the medium and the thermal contact resistance between probe and medium.

Non-steady-state probe methods are widely employed for thermal conductivity measurements in dry or wet porous media. In technical science, for instance, it can be applied to measure the insulating properties as well as the heat storage properties of building materials. In geology, a thorough knowledge of the thermal parameters of the various strata may be obtained by this method, which is useful for studies in earth history, for predicting earthquakes and for research on oil-bearing layers. In agriculture, the thermal parameters of the soil must be determined for studies about the heat balance at the earth's surface, frost penetration and storage of agricultural products and foods.

Thermal conductivity measurements with the probe method have several advantages. The method can be used *in situ*, the measuring time is rather short and, compared with other kinds of thermal conductivity measurement, this method is, to a high degree, non-destructive to the material examined. Apart from these, the possibility to gain simultaneous values of the volumetric heat capacity can be mentioned. However, up to now the accuracy of the determination of this latter physical property has remained poor. In this paper a more reliable simultaneous determination of heat capacity and thermal conductivity is presented.

PRESENT APPROACH

Several mathematical models have been developed to calculate the thermal parameters of the medium. The simple, perfect line source analysis is still often used. However, for dry materials the accuracy is not so high and it is impossible to obtain the volumetric heat capacity and the contact resistance by means of this model.

More accurate results can be obtained from the theoretical approach of Blackwell [1], Jaeger [2] and De Vries and Peck [3]. They all started from a cylindrical model of the measuring probe, which allows one to account for the different thermal properties of the probe and the surroundings. The dimensions of the probe and its components, as well as the contact resistance, were introduced. The mathematical approach of the three models was very similar: they all resulted in a first-order time model. There were only some differences in composition and in the introduction of the various thermal properties of the probe. However, none of these models described a proper way to calculate the thermal conductivity and the heat capacity simultaneously from the temperature response.

References [4, 5] developed a revised model of the probe, based on that of Jaeger and hence called the modified Jaeger model. Using that model, the thermal conductivity λ, the volumetric heat capacity C and the contact resistance Γ (per unit of length) could be

NOMENCLATURE

- a thermal diffusivity of the medium [$m^2\ s^{-1}$]
- C volumetric heat capacity [$J\ m^{-3}\ K^{-1}$]
- $\mathbf{d}$ iteration step in the Gauss–Newton method, $(\delta\lambda, \delta C, \delta\Gamma)$
- F mathematical description of the temperature response; a non-linear function of the state vector $\mathbf{x}$ [K]
- Q heat production per unit length [$W\ m^{-1}$]
- R effective distance between temperature sensor and heating wire [m]
- s radius of the heating wire [m]
- t time
- t_b time of the first measuring point for which the model is valid [s]
- u inside radius of the tube [m]
- v outside radius of the probe [m]
- W internal heat resistance per unit length between the temperature sensor and the outer wall of the probe [$K\ m\ W^{-1}$]
- $\mathbf{x}$ state vector, (λ, C, Γ).

Greek symbols
- Γ (external) contact resistance per unit length [$K\ m\ W^{-1}$]
- Γ_t total heat resistance per unit length [$K\ m\ W^{-1}$]
- θ measured temperature response [K]
- θ^0 theoretical zeroth-order temperature response [K]
- θ' theoretical first-order temperature response [K]
- θ'' theoretical second-order temperature response [K]
- λ thermal conductivity [$W\ m^{-1}\ K^{-1}$]
- τ dimensionless time, $4at/v^2$
- τ' modified dimensionless time
- τ'' second modified dimensionless time
- Φ sum of squares of the differences between model results and measuring points [K^2].

Subscripts
- hw heating wire
- is insulating material
- j iteration step j
- n needle-shaped probe
- tu tube.

determined. Two approaches were considered. If C was previously known, λ and Γ could be calculated accurately to within 1 and 5%, respectively. If C was previously unknown, the method allowed simultaneous determination of λ, C and Γ with accuracies of about 5, 25 and 10%, respectively. A disadvantage of the latter approach was the rather unstable iteration method in which the heat capacity C was determined: only very well performed measuring series could be evaluated in this way.

In this paper the temperature response according to the modified Jaeger model will be worked out in more detail. If the accuracy of the measurements is high enough, a second-order time correction can also be taken into account. Then, using the Gauss–Newton iteration method, it is possible to determine simultaneously the three thermal parameters λ, C and Γ with higher accuracy.

EXPERIMENTAL SET-UP

The most important parts of the needle-shaped probe are a double fold constantan heating wire and a constantan–manganine thermocouple, both with a diameter of 0.1 mm and carefully fitted into a stainless steel envelope. The hot junction of the thermocouple is placed very close to the heating wire; the cold junction is situated at the end of the probe and is assumed to stay at its original temperature. To fix the position of the wires in the cylindrical envelope and to prevent electrical contact, the remaining space is filled with a silicon rubber compound. The length of the probe is about 200 mm and its diameter varies from 1 to 2 mm for different probes. A cross-section of the probe is shown in Fig. 1.

In our case, the temperature response of the probe on a suddenly starting constant energy dissipation inside the probe is measured. For that purpose, a constant electrical heating current is switched on at time $t = 0$ and the temperature response is recorded once per second.

THE PERFECT LINE SOURCE MODEL

To introduce our approach we have to resume the simple perfect line source model that is still used sometimes for the analysis of the temperature response (θ–t curve) [6, 7]. In this model the probe is considered to be an infinitely long, infinitely thin heating wire, embedded in a homogeneous, isotropic medium. The exact solution of this one-dimensional problem is the exponential integral, which can be written on a power expansion of $R^2/4at$, with R the distance between the heating wire and the temperature sensor and a the thermal diffusivity ($a = \lambda/C$). If we use the first two terms of the power series, the temperature response $\theta^0(t)$ is given by

$$\theta^0(t) = A \ln(t) + B \quad (t \gg R^2/4a) \tag{1}$$

where

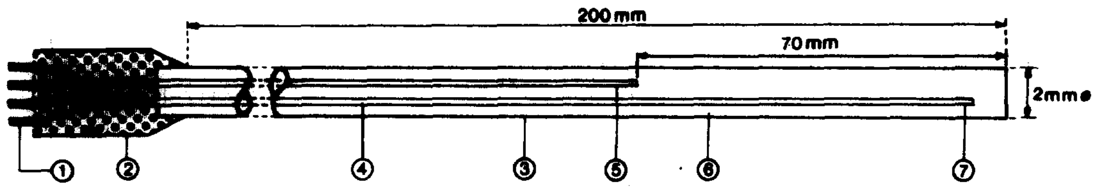

FIG. 1. Axial cross-section of the needle-shaped probe: 1, contacting wires; 2, polyvinyl chloride protector; 3, tube; 4, hot junction of thermocouple; 5, heating wire; 6, insulating material; 7, cold junction of thermocouple.

$$A = Q/(4\pi\lambda) \tag{2}$$

and

$$B = \{Q/(4\pi\lambda)\}\{\ln(4\lambda/CR^2) - C_E\} \tag{3}$$

in which C_E is Euler's constant ($C_E \approx 0.5772$). A big advantage of this simple relation is that the coefficients A and B can be determined by linear regression (a least squares method) and hence λ and C can be calculated.

However, the accuracy is not so good. In particular, the determination of C is unreliable. This is caused by the representation of the probe, which is too simple in this model. A real probe has finite dimensions and finite thermal parameters. Moreover, the temperature rise is measured inside the probe and not directly in the medium. In addition, there will in general be a heat resistance between the temperature sensor and the medium. This parameter consists of two parts: the internal heat resistance W, which accounts for the temperature difference between the temperature sensor and the surface of the probe, and the contact resistance Γ, which accounts for the disturbance in the thermal conductivity of the granular material close to the probe.

THE MODIFIED JAEGER MODEL

As mentioned before, the modified Jaeger model [4, 5] accounts for the dimensions and thermal parameters of the probe and for the thermal contact resistance between probe and medium. The modified Jaeger model was developed in two steps (Fig. 2).

In the first step the real probe was represented by an idealized probe in which the double fold heating wire was replaced by a cylindrical core. Around that core the silicon rubber compound was placed as a coaxial cylinder, enveloped in turn by the outer steel cylinder. The hot junction of the thermocouple was assumed to be infinitely small and its position was defined by the effective radial distance R to the axis of the system.

In the second step, this idealized probe was replaced by a perfectly conducting, massive cylinder with the same radius v as the original probe and with the following definitions:

(a) the volumetric heat capacity of this cylinder is equal to the sum of the heat capacities of the idealized probe

$$C_n = (s^2C_{hw} + (u^2 - s^2)C_{is} + (v^2 - u^2)C_{tu})/v^2; \tag{4}$$

(b) the 'heat resistance' Γ_t between this cylinder and the medium is equal to the real contact resistance Γ and the internal heat resistance W of the probe for $r > R$

$$\Gamma_t \equiv \Gamma + W. \tag{5}$$

Under quasi-steady-state conditions, using the well-known equation for heat conduction in cylinders, we obtain for the internal heat resistance

$$W = \frac{\ln(u/R)}{2\pi\lambda_{is}} + \frac{\ln(v/u)}{2\pi\lambda_{tu}}. \tag{6}$$

De Vries and Peck [3] showed that the steady-state profile is obtained with a good approximation after about 5 s. This is sufficiently small compared with the usual measuring time of about 200 s.

For the modified Jaeger model, the first-order temperature response $\theta'(t)$ can be given by (according to Bruijn *et al.* [5])

$$\theta'(t) = A\ln(t) + B + D/t + E\ln(t)/t \tag{7}$$

where the coefficients A, B, D and E are non-linear functions of heat conductivity λ, heat capacity C and thermal resistance Γ. They are also functions of the probe radius v, the dissipated heat Q, the heat capacity of the probe C_n and the internal heat resistance W. The four latter parameters (v, Q, C_n and W) are considered to be known constants. Of course, they can differ from probe to probe, and Q can differ from measurement to measurement. Coefficients A, B, D and E are given by [5]

$$A = Q/4\pi\lambda \tag{8}$$

$$B = \{Q/4\pi\lambda\}\{\ln(4a/v^2\varepsilon) + 4\pi\lambda(\Gamma + W)\} \tag{9}$$

$$D = \{QCv^2/8\pi\lambda^2\}\{(1 - C_n/C)\ln(4a/v^2\varepsilon) + 1 - (C_n/C)4\pi\lambda(\Gamma + W)\} \tag{10}$$

$$E = \{QCv^2/8\pi\lambda^2\}(1 - C_n/C) \tag{11}$$

where $\varepsilon = \exp(C_E)$.

The modified Jaeger model is valid for time values at which the higher-order terms of the power expansion are much smaller than the sum of the four terms of equation (7). As the first one of these higher-order terms has the order of t^{-2}, we can formulate this condition as

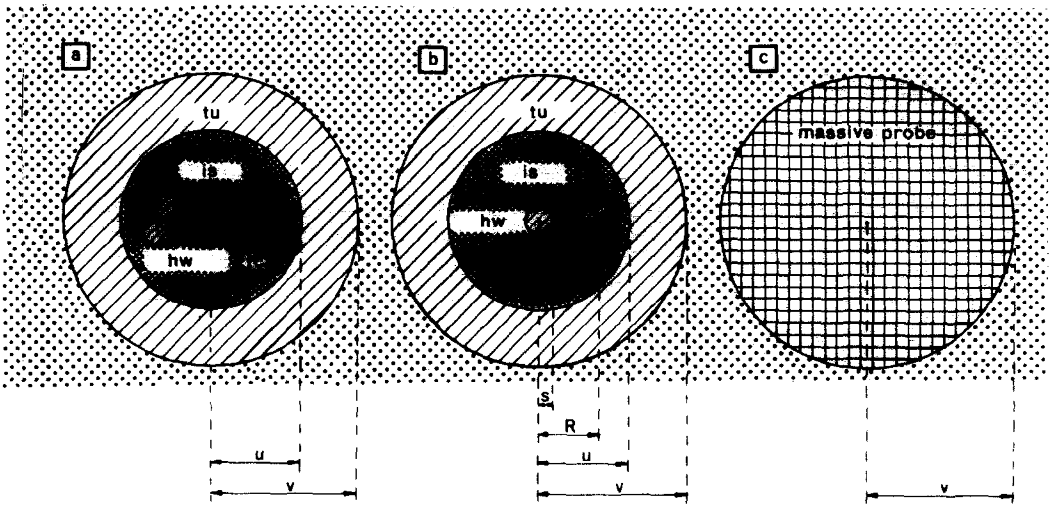

FIG. 2. From probe to modified Jaeger model: (a) real probe; (b) idealized probe; (c) simplified, idealized probe. tu, tube; is, insulating material; hw, heating wire; tc, thermocouple.

$$O(1/t^2) \ll \theta'(t).$$

In working out this condition, we first have to obtain an expression for the second-order time correction $O(1/t^2)$. De Vries and Peck [3] derived the second-order time correction for their model if $r > v$ (outside the probe). We will apply their conclusions about the correction to our model.

We follow De Vries and Peck's notation by introducing the dimensionless time τ such that

$$\tau = 4at/v^2. \qquad (12)$$

With reference to equations (8) and (9), the first two terms of equation (7) can be described in terms of τ

$$A \ln(t) + B = \{Q/4\pi\lambda\}\{\ln(\tau) - C_E + 4\pi\lambda(\Gamma + W)\}. \qquad (13)$$

(This could be considered as a zeroth-order time model.) With a modified dimensionless time τ', equation (13) can be written more simply as

$$A \ln(t) + B = \{Q/4\pi\lambda\} \ln(\tau') \qquad (14)$$

where

$$\tau' = \tau \exp\{4\pi\lambda(\Gamma + W) - C_E\}. \qquad (15)$$

The third and fourth terms of equation (7) are similarly

$$D/t + E \ln(t)/t = \{Q/2\pi\lambda\} \ln(\tau')/\tau(1 - C_n/C). \qquad (16)\dagger$$

Introducing τ'', which is a second modification of the dimensionless time, the modified Jaeger model has the form

$$\theta'(t) = \{Q/4\pi\lambda\}\{\ln(\tau') + 2\ln(\tau'')/\tau\} \qquad (17)$$

where

$$\tau'' = \exp[\tau(D + E\ln(t))/2At]$$
$$\approx \exp[\ln(\tau')/(1 - C_n/C)]. \qquad (18)$$

Both equations (17) and (7) represent the modified Jaeger model, a model with first-order time correction. As remarked before, there is no essential difference between the modified Jaeger model and, for instance, the model of De Vries and Peck. So equations (7) and (17) are also valid for the model of De Vries and Peck; in equation (7) only the coefficients D and E differ slightly, in equation (17) a slightly different definition of τ'' is required [7].

The model of De Vries and Peck describes the temperature response both inside the probe ($r \leqslant v$) and outside the probe ($r > v$). In the latter area a second-order time correction was also derived

$$O(1/t^2) = \{-3Q/4\pi\lambda\}[(1 - C_n/C)\ln(\tau')/\tau]^2 + O(\ln(\tau'/\tau^2)). \qquad (19)$$

Our modified Jaeger model gives the temperature rise inside the probe ($r \leqslant v$) and, in this area only, we must find a second-order time correction. As a consequence of the above-mentioned similarity between the two models and using the results of De Vries and Peck, equation (19), for the second-order time correction, it is possible also to extend the modified Jaeger model with a second-order time correction. This correction provides only the order of magnitude, but that is the only thing we want to know: an expression for the time interval where the modified Jaeger model is valid. Using equation (18), the second-order time correction can be written as

$$O(1/t^2) = \{-3Q/4\pi\lambda\}[\ln(\tau'')/\tau]^2. \qquad (20)$$

† Equation (16) is not entirely correct; looking at equation (10) we see that in the definition of coefficient D the second term equals 1 instead of $4\pi\lambda(\Gamma + W)$. This error is ignored because we are only interested in the order of magnitude of the second-order time correction.

From these results, the time interval for which the modified Jaeger model (17) is valid can be determined

$$|-3\{Q/4\pi\lambda\}[\ln(\tau'')/\tau]^2| \ll \{Q/4\pi\lambda\}[\ln(\tau')+2\ln(\tau'')/\tau]. \quad (21)$$

Considering that $\ln(\tau') \approx \ln(\tau'')$ and supposing that the left-hand side of the inequality is a factor f smaller than the right-hand side, we obtain an expression for the dimensionless starting time τ_b' of the time interval

$$\tau_b' = -1+\sqrt{(1+3f\ln(\tau_b'))}. \quad (22)$$

This expression can be solved by a numerical method; for instance by the successive substitution method [8]. It is quite simple now to obtain the (normal) starting time of the time interval

$$t_b' = \tau_b' v^2/4a.$$

The second-order term, equation (20), can be seen as an extension of the modified Jaeger model, but it must be stressed that it will be an approximation. The term is relatively small (if $\tau > 1$) and decreases rapidly for increasing time. With this term the second-order time model for the temperature response can now be written as

$$\theta''(t) = \{Q/4\pi\lambda\}[\ln(\tau')+2\ln(\tau'')/\tau - 3(\ln(\tau'')/\tau)^2] \quad (23)$$

in which the model error is of the order of magnitude of $[\{3Q/4\pi\lambda\}[\ln(\tau'')/\tau^2]$. The use of the extended model is only possible if

$$\{Q/4\pi\lambda\}[\ln(\tau'')/\tau^2] \ll \{Q/4\pi\lambda\}[\ln(\tau') + 2\ln(\tau'')/\tau]. \quad (24)$$

Considering again that $\ln(\tau') \approx \ln(\tau'') > 0$ and that the right-hand side of the inequality is at least a factor g bigger than the left-hand side, it is possible to calculate the dimensionless starting time

$$\tau_b'' = -1+\sqrt{(1+3g)}, \quad \text{with } g \approx f \quad (25)$$

whereas the starting time for the second-order time model is

$$t_b'' = \tau_b'' v^2/4a.$$

The modified Jaeger model with second-order time correction provides a better description of the temperature response in the time interval $[t_b'', t_b]$. In this time interval the response is relatively steep. The temperature increase in the immediate vicinity of the hot junction of the thermocouple is relatively large (some several cK s^{-1}). In this time interval a lot of heat is stored in the probe itself and in the medium close to it. A good description of the temperature response in this interval is therefore important if we want to determine the heat capacity of the medium. The more accurate we can describe this part, the better we will evaluate the heat capacity of the medium.

GAUSS–NEWTON ITERATION

The modified Jaeger model with second-order time correction describes the temperature response quite accurately. On the other hand, the calculation of the parameters λ, C and Γ from the temperature response is complicated. As stated before, the mathematical description of the model is a non-linear function of the thermal parameters of the probe and the medium. The thermal parameters of the probe are known from calibration, whereas the two thermal parameters of the medium (λ and C) and the contact resistance Γ are unknown. This set of three parameters is what we want to determine. We define the state vector $\mathbf{x}$ as the set of three thermal parameters: $\mathbf{x} = (\lambda, C, \Gamma)$. Rewriting the second-order time model (23) as a function F of $\mathbf{x}$ and t and eliminating at the same time τ' with equation (15) and τ'' with equation (18), it follows that:

$$F(\mathbf{x},t) = A(\lambda)\ln(t) + B(\lambda, C, \Gamma) + [D(\lambda, C, \Gamma) + E(\lambda, C)\ln(t)]/t - 3[D(\lambda, C, \Gamma) + E(\lambda, C)\ln(t)]^2/[4A(\lambda)t^2] \quad (26)$$

if $t \geqslant t_b''$ and where $A(\lambda)$, $B(\lambda, C, \Gamma)$, $D(\lambda, C, \Gamma)$ and $E(\lambda, C)$ are non-linear functions of the state vector $\mathbf{x}$. They are defined in the modified Jaeger model of equations (8)–(11).

The temperature response contains $t_b''+n$ measuring points. For each of these measuring points we want to apply equation (26) (if $t > t_b''$). This gives n equations of the kind $\theta(t) = F(\mathbf{x}, t)$: a set of n equations with only three unknown parameters. This set has to be solved with a least squares method. To that end we minimize the sum of squares Φ as a function of the state vector $\mathbf{x}$, where Φ is defined as

$$\Phi(\mathbf{x}) = \sum_{t=t_b''}^{t_b''+n} (F(\mathbf{x},t) - \theta(t))^2. \quad (27)$$

If we minimize Φ, its derivatives must be zero, so

$$\mathrm{d}\Phi(\mathbf{x})/\mathrm{d}\mathbf{x} = \mathbf{0}. \quad (28)$$

Equation (28) results in three sub-equations

$$\partial\Phi(\mathbf{x})/\partial\lambda = 0; \quad \partial\Phi(\mathbf{x})/\partial C = 0 \quad \text{and} \quad \partial\Phi(\mathbf{x})/\partial\Gamma = 0. \quad (29)$$

This set of three equations contains the three unknown parameters λ, C and Γ. They could be solved straight away if they were linear. Dealing with non-linearity with respect to the state vector, we need to take the first-order approximation of the temperature response $F(\mathbf{x}, t)$. Here we will use the well-known Newton linearization, which is allowed if none of the three derivatives $\mathrm{d}F(\mathbf{x})/\mathrm{d}\mathbf{x}$ equals zero in the neighbourhood of $\mathbf{x}$.

Starting with a known state vector $\mathbf{x}_0$ in the neighbourhood of the desired state vector $\mathbf{x}$, we can write as a first approximation

$$F(\mathbf{x}_0, t) = F(\mathbf{x}, t) - [\mathrm{d}F(\mathbf{x}, t)/\mathrm{d}\mathbf{x}, (\mathbf{x}-\mathbf{x}_0)] \quad (30)$$

where the second term of the right-hand side is the inproduct of two vectors, which equals

$$\partial F(\mathbf{x})/\partial\lambda(\lambda-\lambda_0)+\partial F(\mathbf{x})/\partial c(C-C_0)+\partial F(\mathbf{x})/\partial\Gamma(\Gamma-\Gamma_0).$$

Because we chose $\mathbf{x}_0$ in the neighbourhood of $\mathbf{x}$, it can be assumed that the derivatives of the model function F are almost equal in both points

$$\mathrm{d}F(\mathbf{x},t)/\mathrm{d}\mathbf{x}\approx \mathrm{d}F(\mathbf{x}_0,t)/\mathrm{d}\mathbf{x}_0$$

so

$$F(\mathbf{x}_0,t)=F(\mathbf{x},t)-[\mathrm{d}F(\mathbf{x}_0,t)/\mathrm{d}\mathbf{x}_0,\mathbf{d}] \quad (31)$$

with

$$\mathbf{d}=\mathbf{x}-\mathbf{x}_0.$$

Writing $F(\mathbf{x},\mathbf{t})$ in equation (31) explicitly, we see that it is a non-linear function of the known state vector $\mathbf{x}_0$, and at the same time it is a linear function of the difference state vector $\mathbf{d}=(\lambda-\lambda_0, C-C_0, \Gamma-\Gamma_0)$. The difference state vector $\mathbf{d}$ can be solved from equations (31) and (27)

$$\Phi(\mathbf{x})=\sum_{t=t_0''}^{t_0''+n}\{F(\mathbf{x}_0,t)+[\mathrm{d}F(\mathbf{x}_0,t)/\mathrm{d}\mathbf{x}_0,\mathbf{d}]-\theta(\mathrm{t})\}^2. \quad (32)$$

Now, the sum $\Phi(\mathbf{x})$ is linear with respect to the state vector. It has to be minimized according to equation (28). This can be carried out with the Householder transformation [8], which is widely employed for multiple regression analysis.

Starting from $\mathbf{x}_0$, a better solution for the state vector $\mathbf{x}$ is

$$\mathbf{x}_1=\mathbf{x}_0+\mathbf{d} \quad (33)$$

where $\mathbf{x}_1$ is the first iteration step. The jth iteration step is given by

$$\mathbf{x}_j=\mathbf{x}_{j-1}+\mathbf{d}_j. \quad (34)$$

The difference state vector $\mathbf{d}_j$ must be calculated for each step j using equation (29) (replacing $\mathbf{x}_0$ by $\mathbf{x}_{j-1}$) and the Householder transformation. The stop criterion of the iteration is determined by the sum of squares $\Phi(\mathbf{x}_j)$. When $\Phi(\mathbf{x}_j)$ does not get any smaller the iteration must be finished. This iteration method is well known as the Gauss–Newton method.

A disadvantage of this method is that the first try for the state vector must be chosen near to the real value (the value to be calculated). The iteration method can only be used in a small area in the vector space, because outside this area the method loses its square convergence and only a very slow linear convergence remains. It is even possible that no global convergence will take place at all; the method will then reach a local minimum in $\Phi(\mathbf{x})$ and a wrong value of $\mathbf{x}$ will be calculated.

To enlarge the convergence area of the iteration method (in other words, to enlarge a possible choice of $\mathbf{x}_0$), relation (34) can be extended as follows:

$$\mathbf{x}_j=\mathbf{x}_j+f_j\mathbf{d}_j \quad (35)$$

where f_j is a fitting constant.

This is called the Gauss–Newton method with line minimization [8]. For each iteration step j, f_j is determined to obtain the best $\mathbf{x}_j$. The best $\mathbf{x}_j$ is of course the state vector in which $\Phi(\mathbf{x}_j)$ reaches its minimum. (It must be noticed that attention still has to be paid to avoid local minima.)

CALIBRATION MEASUREMENTS

To evaluate the theory developed in the previous sections, calibration measurements were performed for several temperatures. To that end, a measuring cylinder filled with the calibration material and supplied with a probe along its axis was placed into a thermostatic bath. The temperature of the bath was controlled with a stability of less than 0.01 K.

Because the mathematical model requires very well-defined measuring conditions, the calibration material has to be as homogeneous and isotropic as possible. For that reason a dilute agar gel of 0.4 mass percent was used, which has thermal parameters almost equal to those of pure water. The heat capacity of water is 4.19 MJ $\mathrm{m}^{-3}\,\mathrm{K}^{-1}$ [10], and the thermal conductivity to a good approximation (Powell *et al.* [9]) can be represented by $\lambda=0.560+19\times10^{-3}\,T$, with T in °C. As a consequence of the zero contact resistance Γ between the probe and the medium, the measured total heat resistance Γ_t now equals the internal heat resistance W. This heat resistance W is a probe property, which will be used in future measurements to determine the real contact resistance Γ in the case of an arbitrary porous medium. The thermal parameters λ, C and Γ_t are calculated using the methods of the previous sections.

To avoid electromagnetic noise from the thermostat, we switched off the temperature control during the measurements themselves. However, temperature drift in the thermostatic bath could now result. In particular, if the temperature difference between the measuring cylinder and the room increases, this temperature drift also increases. To investigate the influence of this drift, two different measuring series were performed. First, the series without drift correction was carried out with the probes numbered as 19, 25 and 26. Second, the series with the drift correction was carried out; here, the probes numbered as 20 and 24 were used.

Figure 3 shows that the measured thermal conductivities agree well with the above-mentioned literature values: all lines are parallel to each other with relative distances up to 3%. In particular, probes 26 (△) and 20 (●) are very close to the measurements of Powell *et al.* Apart from this, all measurements have relative errors of about 0.3%. The results of Bruijn *et al.* [5] are systematically about 3% higher

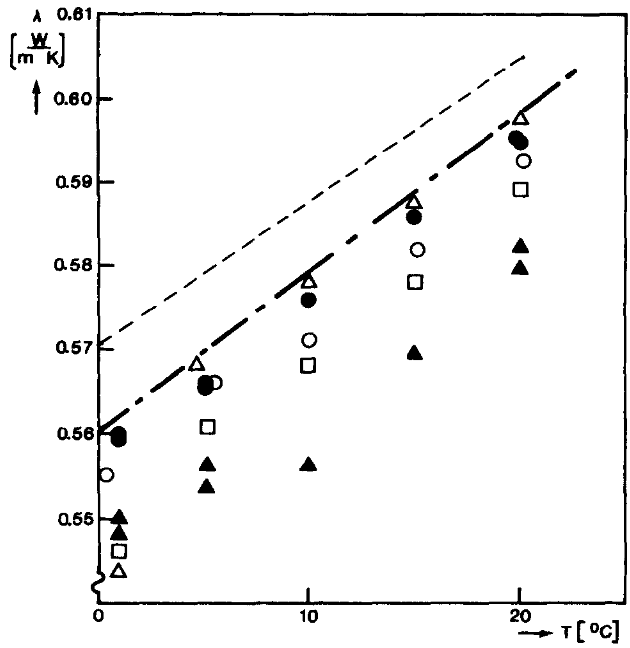

FIG. 3. Thermal conductivity λ as a function of the temperature T measured with different probes: ○, probe 19; ●, probe 20; ▲, probe 24; □, probe 25; △, probe 26; ———, literature value [9]; – – –, literature value [5].

than our values. As pointed out before, both methods are based on the modified Jaeger model. The differences between the two methods are:

(a) the second-order time correction which has been added to our model;

(b) another iteration method to calculate the thermal parameters.

As can be seen, the accuracy of the first series (outlined signs) is less at smaller temperatures—a consequence of the increasing temperature difference between the cylinder and the ambient room. In the second series the accuracy is constant over the total temperature range. In this series the temperature drift was determined immediately before a measurement was carried out. Afterwards, a correction for this drift (between 0.1 and 0.3 nV s^{-1}) was applied.

For the measured heat capacities (Fig. 4) conclusions similar to those for the thermal conductivities can be drawn. The errors in C, however, are high compared with the errors in λ; the relative errors are about ten times bigger. This can be explained as follows: for the calculation of C we need at least the first-order time correction of the temperature response, $(D+E\ln(t))/t$. This term is much smaller than the temperature response itself; hence the relative error in this term will strongly influence the value of C. When the second-order time correction is added, the sum of the first- and second-order time corrections is bigger (mainly because we apply the interval for smaller time values), so the value of C will be more accurate. But still the accuracy is essentially less than the accuracy in λ. Finally, the total heat resistance $\Gamma_t = \Gamma + W$ is discussed. In agar gel the contact resistance can be considered zero (perfect contact between the gel and the probe), so the total heat resistance equals the internal resistance W (equation (5)). This method is used to calibrate W. For measurements in materials with a contact resistance Γ (for instance, porous materials) this calibrated value for W is subtracted from the calculated total heat resistance Γ_t to obtain a value for Γ.

It is clear that the internal heat resistance is a property of the probe itself. It changes from probe to probe (Fig. 5). As can be seen, the relative error is less than 3%. Again, a slight influence of the temperature drift can be observed (probes 25 and 26). From the internal heat resistance it is possible to calculate an effective radius R (see equation (6) and Fig. 2(b)). This radius can be seen as the effective distance between the heating wire and the hot junction of the thermocouple inside the probe.

CONCLUSIONS

The modified Jaeger model provides the physical basis for describing the temperature response of a needle-shaped probe in a medium. If a second-order time correction is added, the steeper part of the temperature response is also described accurately. This part is important for the calculation of the heat capacity of the medium.

To obtain the three desired thermal parameters (conductivity, heat capacity and contact resistance), a

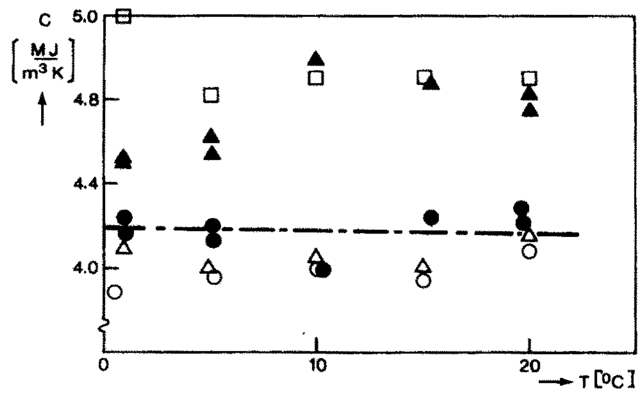

FIG. 4. Heat capacity C as a function of the temperature T measured with different probes: ○, probe 19; ●, probe 20; ▲, probe 24; □, probe 25; △, probe 26; — — —, literature value [10].

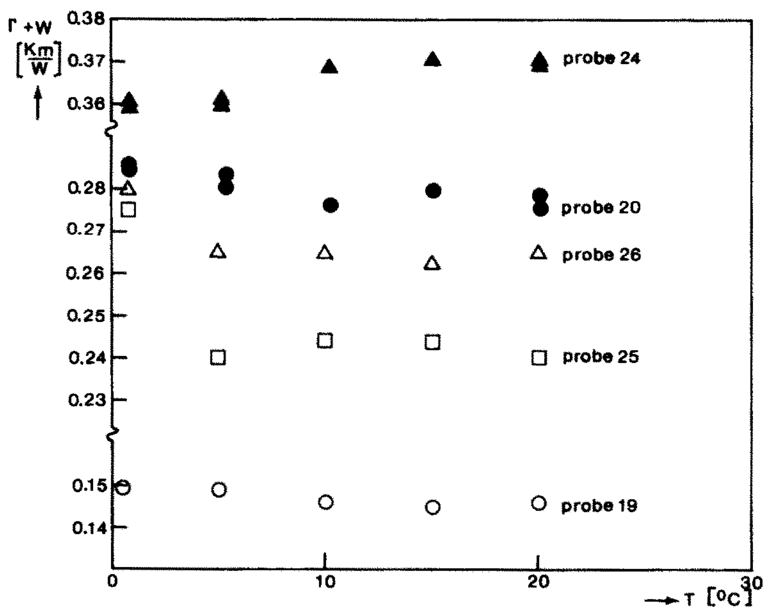

FIG. 5. Total heat resistance Γ_t as a function of the temperature T measured with different probes: ○, probe 19; ●, probe 20; ▲, probe 24; □, probe 25; △, probe 26.

least squares method can be applied: the Householder transformation (for multiple regression analysis) in combination with the Gauss–Newton iteration (for dealing with nonlinearity).

The iteration method has been successfully applied in a numerical model called 'NAALD' [7]. Results of measurements in agar gel with randomly chosen probes show that the inaccuracy in the thermal conductivity λ is less than 3%, the estimation of the volumetric heat capacity C is accurate to within 5–25% and the total heat resistance Γ_t can be determined with a relative error of about 3%. Furthermore, it can be remarked that not all probes give the same results. This probe dependence in the accuracy of λ and C is probably caused by small deviations in the construction. The very thin thermocouple wires, as well as the heating wire, always tordate and bend a bit when they are fitted into the stainless steel envelope. This phenomenon was shown on p. 64 of ref. [4]. To obtain better results, especially for the heat capacity, the probes have to be constructed carefully, and they have to be selected on the basis of their physical cylinder symmetry (by taking X-ray photographs).

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UN NOUVEAU MODELE POUR LA METHODE DE LA SONDE EN REGIME VARIABLE POUR MESURER LES PROPRIETES THERMIQUES DES MILIEUX POREUX

Résumé—Le point de départ est le modèle de Jaeger modifié. Ce modèle cylindrique prend en compte les propriétés principales de la sonde, la conductivité thermique et la chaleur massique du milieu et la résistance de contact entre la sonde et le milieu. La description mathématique conduit à des fonctions non linéaires de ces trois paramètres. On utilise la méthode d'iteration de Gauss–Newton. Pour obtenir une meilleure description de la mesure pour des petites valeurs du temps, on ajoute une correction de second ordre du temps. Les résultats d'un essai expérimental donnent une imprécision de 0,5 à 2% pour la conductivité thermique, tandis que les deux autres paramètres sont précis à 5–15% près.

EIN NEUES MODELL FÜR DAS INSTATIONÄRE VERFAHREN ZUR MESSUNG THERMISCHER EIGENSCHAFTEN PORÖSER STOFFE

Zusammenfassung—Ausgangspunkt ist das modifizierte Jaeger-Modell. Dieses zylindrische Modell berücksichtigt die Haupteigenschaften der Probe, die Wärmeleitfähigkeit und die Wärmekapazität des Mediums und den thermischen Kontaktwiderstand zwischen Probe und Medium. Da die mathematische Beschreibung zu nichtlinearen Funktionen dieser drei Parameter führt, wird die Iteration nach Gauss–Newton angewandt. Um eine bessere Beschreibung der Messungen für kleine Zeitschritte zu erhalten, wird eine Zeitkorrektur zweiter Ordnung hinzugefügt. Die Untersuchungsmethode hat einen Fehler von 0,5 bis 2% bei der Wärmeleitfähigkeit, während die Genauigkeit bei den anderen beiden Parametern zwischen 5 und 15% liegt.

НОВАЯ МОДЕЛЬ ДЛЯ РАЗРАБОТКИ НЕСТАЦИОНАРНОГО МЕТОДА ИЗМЕРЕНИЯ ТЕПЛОФИЗИЧЕСКИХ ПАРАМЕТРОВ ПОРИСТЫХ СРЕД

Аннотация—Для разработки метода используется модифицированная модель Егера. В этой цилиндрической модели учитываются основные характеристики датчика, теплопроводность и теплоемкость среды, а также контактное сопротивление между датчиком и средой. Так как математическая формулировка содержит нелинейные функции от этих трех тепловых параметров, используется итерационный метод Гаусса–Ньютона. Для получения более точных результатов измерений при небольших значениях времени добавляется временная поправка второго порядка. Экспериментальные результаты по теплопроводности получены с точностью 0.5–2%, а остальным двум параметрам — с точностью до 5–15%.